Exploring Students' Argumentation Ability in Mathematical Problem Solving Based on Levels of Productive Struggle

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Abstract
Mathematical argumentation is an essential ability in problem-solving; however, students' ability to construct arguments is often influenced not only by cognitive factors but also by affective aspect. The research project investigates how 11th grade students develop mathematical argumentation ability through their different levels of productive struggle. The study uses a qualitative research design which uses case study methodology. The study used three data collection methods which included mathematical argumentation ability tests and productive struggle questionnaires and interviews. Three students were selected through purposive sampling to represent high and moderate and low productive struggle categories. The researchers collected data through mathematical argumentation tests and productive struggle questionnaires and interviews, which they used to analyze the data according to four argumentation aspects that McNeill and Krajcik identified as claim, evidence, reasoning and rebuttal. The research results demonstrated that students with high productive struggle constructed complete arguments while students with moderate productive struggle created partial arguments that contained reasoning errors. The students with low productive struggle could not complete all argumentation tasks because they had not learned the required concepts and did not have enough determination to overcome challenges. These findings confirm that mathematical argumentation ability are not only influenced by cognitive aspects but also by productive struggle as an affective aspect that determines students' resilience in solving problems.

1. Introduction

The study of mathematics is a fundamental discipline which develops critical thinking and creative thinking abilities that people require to tackle real-world challenges. Students who study mathematics in school acquire the skills to think logically and systematically while developing their critical thinking ability and learning to work with others (Monteleone et al., 2023). The National Council of Teachers of Mathematics (NCTM, 2000), states that students must develop reasoning skills as a fundamental competency which they need to achieve. Afifah et al. (2024) define reasoning ability as an essential competency which students must acquire. Reasoning enables students to understand mathematical structures, formulate generalization, and justify their ideas (Ulfiyati et al., 2025). In this context, mathematical argumentation is an important ability because it reflects how students express their reasoning through statements, proofs, and justifications (Hanna, 2020).

Despite its importance, students' mathematical argumentation ability is often not optimally developed. Many students are able to obtain answers but struggle to justify their reasoning or construct valid arguments. Reasoning ability is closely related to mathematical argumentation, as argumentation represents the externalization of students' interval reasoning processes (Bredow & Knipping, 2025; Stylianides, 2009). Through argumentation, students are able to make conjectures, evaluate arguments, and provide justifications based on evidence (Martins et al., 2023). Therefore, argumentation is seen as a form of expression of higher-order thinking processes related to reasoning and communication (Lehmann & Friend, 2025). However, in practice, students often experience difficulties in constructing valid arguments and providing appropriate justification, indicating that their argumentation ability is not yet optimally developed (Admoko et al., 2021; Muhtadi, 2022).

Mathematical argumentation ability refers to the process of constructing claims supported by evidence and reasoning (Bieda & Staples, 2020; Parico et al., 2020). This process involves providing explanations or justifications for a claim, either individually or collaboratively. Argumentation is also closely related to communication skills, as students need to express mathematical ideas through symbols, representations, and written or verbal explanations (Arifin et al., 2025; R. N. Lubis et al., 2023). Effective communication enables

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students to organize and clarify their mathematical thinking and evaluate the arguments of other (Novianti, 2025; Uzun, 2024). Students need to develop their argumentation ability which will help them complete this stage that starts with problem comprehension and ends with official solution presentation (Hidayat & Prabawanto, 2018; Lubis & Lubis, 2024). Argumentation ability involves the ability to present (critical) reasoning accompanied by adequate data and theoretical support for a mathematical (logical) problem (Subkhi et al., 2025). According to Lee and Lai (2024), argumentation ability can train students to be free in deciding the solution to a problem.

In addition, cognitive aspect, affective aspect also need to be considered. One important affective aspect that students need to have is productive struggle. Productive struggle refers to students' effort to understand and solve problems through persistence and deep thinking (Salazar, 2022). The program helps students acquire fresh knowledge while they develop their understanding and problem-solving skills (Angelina et al., 2023; Melani et al., 2023). The process of productive struggle requires students to stay focused on their difficult work while they test how well their chosen methods function (Listiawati, 2016; Nainggolan et al., 2024; Samosir et al., 2023). In this context, productive struggle is assumed to support students' argumentation ability, as constructing arguments requires sustained effort, careful reasoning, and evaluate of strategies. Students need continuous chances to develop their mathematical skills through productive struggle which we must support throughout their learning process. The study of mathematics needs both concept understanding and relationship knowledge for students to achieve mastery (NCTM, 2000). Thus, productive struggle can help students understand and search deeply for solutions to problems in accordance with the mathematical concepts used. Productive struggle certainly supports argumentation ability as a cognitive aspect that requires a high level of effort to acquire. When students encounter difficulties in solving a problem, they need to be careful in choosing and evaluating the strategies used and be prepared to abandon strategies that do not lead to acceptable solutions.

Although many studies have examined argumentation ability, research examining mathematical argumentation ability based on students' productive struggle remains limited. The research performed by Putra et al. (2022) studied mathematical argumentation ability through examination of students' resilience based their abilities to create mathematical arguments. The research needs to be done because it requires investigation of mathematical argumenting ability which depend on students' ability to handle productive struggle. Based on this description, the study aims to analyze that mathematical argumentation ability based on their level of productive struggle. The results of this study will describe the level of student argumentation based on the level of productive struggle.

2. Method

2.1. Study Design

The study aimed to investigate how students develop their mathematical argumentation skills when they experience different levels of productive struggle. The study implemented a qualitative research design which used a case study method for its investigation. A case study research method in which researchers explore a specific phenomenon (case) at a given time and activity and collect detailed and in-depth information using various data collection procedures over a specific period (Assyakurrohim, 2022; Creswell & Creswell, 2023).

2.2. Participants

The participants in this study were 11th grade students of one of the Senior High School in Garut Regency, West Java in the 2024/2025 academic year, totaling 33 students. The students were generally aged between 16-17 years and consisted of 13 male and 20 female students. All participants had studied the topic of System of Linear Equations in Three Variables.

The researchers used purposive sampling to select three students who each represented different levels of productive struggle (high, moderate, and low) based on the data that had been collected from the written test and questionnaire results. The selection criteria included: (1) students' scores on the productive struggle questionnaire, (2) their performance on the mathematical argumentation test, and (3) their ability to communicate their reasoning during interviews. According to Sugiyono (2018), purposive sampling is based on certain considerations, which are designed to help researchers explore and understand the conditions of the research object in depth.

2.3. Instruments

In this study, the researcher acted as the primary instrument, as is common in qualitative research, being directly involved in collecting, analyzing, and interpreting the data. The supporting instruments used in this study consisted of mathematical argumentation ability test, a productive struggle questionnaire, and interview guidelines.

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The mathematical argumentation test consisted of two essay items designed to assess students' argumentation ability in solving mathematical problems. The test instruments were developed based on argumentation indicators proposed by McNeill and Krajcik (2011) namely claim, evidence, reasoning, and rebuttal. To ensure the instrument's suitability, a content validity test was conducted through expert evaluation by two validators, consisting of mathematics education lecturer and high school mathematics teacher. In this study, validation was conducted on three main aspects: alignment with objectives (argumentation indicators), content quality, and construction and language. Based on the validation results, the two validators assessed that the two test items met the validity criteria regarding objectives, content, and construction and language; therefore, the instrument was deemed suitable for use.

The productive struggle questionnaire was adapted from Masitoh (2025), which has been previously validated and reported to have acceptable reliability (Cronbach's alpha = 0.854). The questionnaire consists of 30 items comprising positive and negative statements aligned with the indicators of productive struggle developed by Warshauer (2015), namely asking questions, encouraging, giving time, and persevering.

The interview guidelines were used in semi-structured interviews, with a set of question outlines based on the questions administered during the test. The interviews were conducted to corroborate the survey results regarding students' productive struggle and their mathematical argumentation ability as demonstrated through the test questions. The interviews were conducted with a student's representing each level of productive struggle.

2.4. Data Analysis

The Miles and Huberman framework for data analysis requires three steps which include data reduction and data presentation and conclusion drawing (Miles & Huberman, 1984). The researchers will begin data reduction by selecting essential information and removing nonessential data which will assist them in reaching their research conclusions. The mathematical argumentation ability test results will be shown through a short explanation that includes supporting evidence. Before drawing conclusions, researchers will re-examine the data that has been presented previously and look at the relationship between mathematical argumentation ability and productive struggle. After seeing the relationship between the two, researchers will conclude the research results clearly and transparently in accordance with the available data. It is suggested that group reorganization be based upon levels of productive struggle and will be revealed from the data shown in Table 1 (Mefiana et al., 2024).

Table 1. Student Grouping Based on Productive Struggle

Criteria	Category
PS>80	High
65≤PS≤80	Moderate
PS<65	Low

After grouping students' productive struggle levels, one student from each level was selected to conduct an in-depth analysis on students' mathematical argumentation ability in answer test questions on System of Linear Equations in Three Variables. Indicators of argumentation ability based on McNeill and Krajcik (2011) and the levels of argumentation is adapted from McNeill and Krajcik (2008) are shown in Table 2.

Table 2. Indicators and Levels of Argumentation Ability Based on McNeill & Krajcik

Aspect	Level		
	0	1	2
Claim: The original question received its answer through the conclusion.	The statement fails to make a proper claim because it presents false information about the subject.	The statement about the subject is correct but does not provide complete information.	The statement about the subject is both correct and complete.
Evidence: Scientific data which supports the claim but the data needs to meet both suitable and adequate standards to establish the claim.	The evidence demonstrates absence because it either provides no proof or presents evidence that does not support the claim.	The evidence presented is suitable for the claim but it does not provide enough proof to support the assertion. The evidence may contain some elements which should not be included.	The evidence presented from the evidence base meets both the appropriate and sufficient standards to establish the claim.
Reasoning: A proof which connects the claim to its supporting evidence.	The presentation contains no reasoning or presents reasoning which fails to connect evidence with the claim.	The person provides the needed reasoning which connects the claim with its proof. The person repeats the proof while showing some scientific principles	The person presents reasoning which connects the evidence to the claim. The scientific principles presented by the person meet both the required

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Aspect	Level		
	0	1	2
		which do not meet the required level of evidence.	standards and the needed scientific standards.
Rebuttal: Acknowledging and rejecting counterclaims with evidence and reasoning showing when they do not apply.	The argument fails to present a counterclaim because the author does not provide evidence to disprove the opposing argument.	The writer presents a counterclaim but fails to show enough proof and logical arguments needed to disprove it.	Provides counterclaim and providing evidence and reasons to reject it.

To obtain a more in-depth mathematical argumentation of the subject, interviews were conducted on data processing and subject responses. The mathematical argumentation analysis result were established through three tiers of productive struggle analysis which included high, moderate, and low productive struggle levels.

3. Results and Discussion

3.1. Results

The research gathered information through three different assessment methods which included mathematical argumentation ability tests and productive struggle questionnaires and student interviews. The productive struggle questionnaires which all students completed were divided into three categories which included high, moderate, and low categories. The categorization of the results of the students' productive struggle questionnaires is presented in Table 3.

Table 3. Score Category Criteria for Students' Productive Struggle

Criteria	Category	Total	Percentage (%)
PS>80	High	8	24.2
65≤PS≤80	Moderate	14	42.4
PS<65	Low	11	33.3

Based on Table 3, most students had moderate productive struggle (42.4% or 14 students). Meanwhile, there are 11 students with low productive struggle, or 33.3%. There are 8 students with high productive struggle, or 24.2%. The next step was to select 3 students, each representing high, moderate, and low productive struggle, to obtain the level of students' mathematical argumentation ability based on tests and interviews, as presented below:

3.1.1. Subject with High Productive Struggle

The results of the mathematical argumentation ability test for students categorized as having high productive struggle (ST) on question number 1 can be seen in Figure 1. In Figures 1, subject ST is able to make complete and accurate claims. Subject ST is also able to provide evidence in the form of data or information obtained from the question so that the evidence can be linked to the calculation process as a reason to support the claim made. The first calculation process carried out by subject ST is the elimination of equations 2 and 3 to produce the variable x first, thereby facilitating the subsequent calculation process. In addition, subject ST is able to provide a counterclaim and sufficient evidence and reasons to reject the counterclaim given in question number 1. The answers to question number 2 are shown in Figures 2.

The controlled test indicated that the subject enjoyed the ability to utilize appropriate and sufficient evidence to complement his reasons for growth to establish arguments. The ST subject used the substitution method for the calculation process. The ST subject was also able to make counterclaims with sufficient and appropriate reasoning and evidence to refute the counterclaims in the questions.

Based on this description, subject ST is able to solve problems accurately and smoothly. Subject ST did not experience any difficulties and was confident in each answer provided. Subject ST successfully proved all their answers through supporting evidence and explanations. ST showed they understood system of linear equations in three variables through their performance. Subject ST, with their high level of productive struggle, was able to provide and fulfill the components of claim, evidence, reasoning, and rebuttal well. Subject ST was able to provide counterclaims and sufficient evidence or reasons to refute the counterclaims provided.

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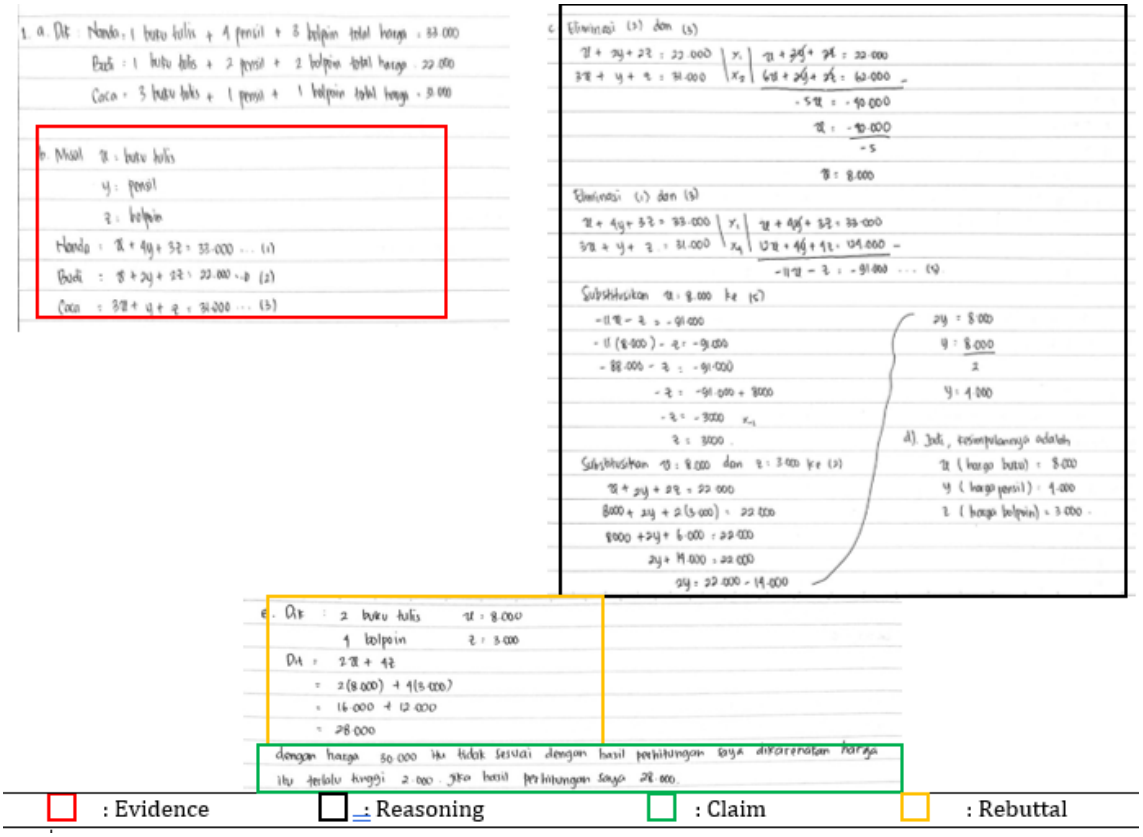

Figure 1. Subject ST's Answers to Equation Number 1

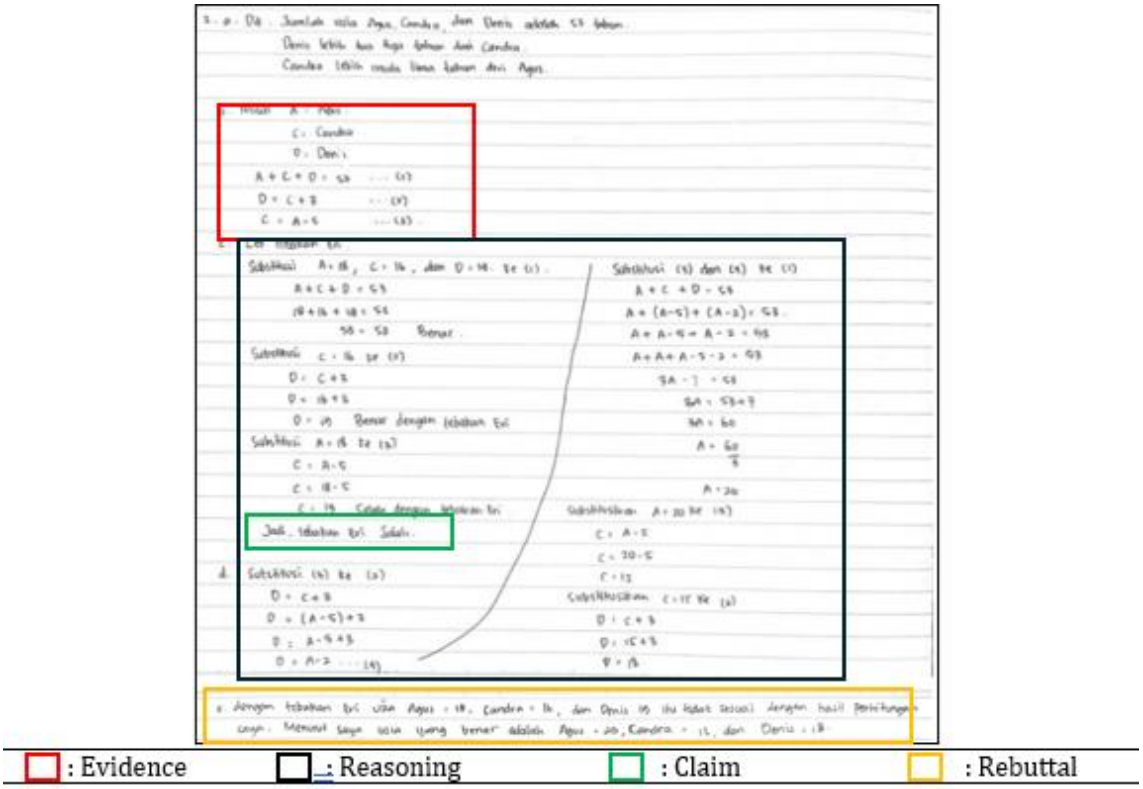

Figure 2. Subject ST's Answers to Equation Number 2

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Further interviews regarding productive struggle were conducted with subject S, and it can be concluded that subject ST often asks teachers or friends to help explain material or questions that subject ST finds confusing and does not understand. Table 4 presents the argumentation ability of ST subjects who display high productive struggle according to McNeill and Krajcik's theoretical framework.

Table 4. Subject ST's Level of Argumentation Ability

No	Aspect	Description	Level
1.	Claim	ST can make accurate and complete claims	2
2.	Evidence	ST can provide evidence through data and information which they collected during the question process. This evidence, according to the requirement, meets the standard of being adequate and suitable for proving the assertion.	2
3.	Reasoning	ST can provide reasons in the form of clear and precise calculation steps from the evidence obtained to support the claim.	2
4.	Rebuttal	ST can determine counterclaims and provide evidence and reasoning to refute counterclaims.	2

3.1.2. Subject with Moderate Productive Struggle

The results of the mathematical argumentation ability test for students categorized as having moderate productive struggle (SS) on question number 1 can be seen in Figure 3.

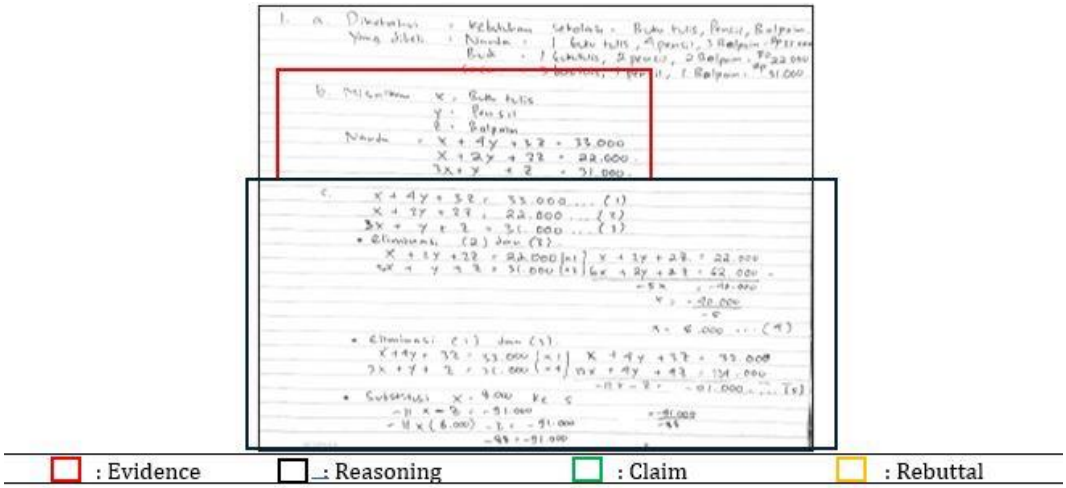

Figure 3. Subject SS's Answers to Equation Number 1

From the Figure 3, it is to be noted that SS has not been able to give the final result clear and certain. Subject SS can provide evidence in the form of data or information known in the question. However, subject SS is unable to provide a correct and complete calculation as evidence because there is an error in the process of substituting values into equation 4, so subject SS is unable to solve the problem. In addition, subject SS is not yet able to provide a counterclaim with sufficient and appropriate reasoning and evidence to reject the counterclaim given in the question. The answer to question 2 displayed in Figure 4.

It could be extrapolated from the comparison that the subject SS merely records the data or information mentioned in the question while the other subject behaves equally in both cases. Subject SS tends to be able to make claims, but the claims made are incorrect. Subject SS is unable to provide enough valid reasons to support his claim in question number 2. Subject SS can only provide reasons based on Eri's guesses, which are inaccurate. In addition, subject SS is also unable to provide a counterclaim to the claim made in question number 2.

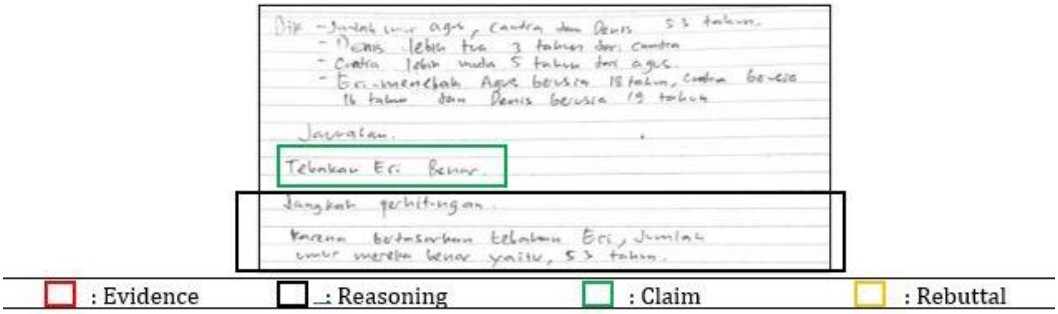

Figure 4. Subject SS's Answers to Equation Number 2

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Based on this description, subject SS is not yet able to provide a complete claim. Subject SS still has difficulty in making reasoning and rebuttals. Subject SS can provide a claim, but it is incomplete. Subject SS cannot provide sufficient explanation to support his claim. Subject SS can determine the information in question number 1 well and completely. However, in question number 2, subject SS is not yet able to determine the information completely. In question number 1, subject SS can provide an explanation, but it is incomplete because there is an error in the calculation so that the calculation process cannot be continued properly. Meanwhile, in question number 2, it appears that subject SS is confused in working on the question because there is an equation that only contains two variables.

Further interviews were conducted with subject SS regarding productive struggle. Subject SS was sometimes hesitant and embarrassed to ask teachers questions about things they did not understand. Subject SS felt more comfortable asking friends about material or questions they did not understand. However, subject SS was highly motivated to try to solve problems, so they would try to face difficulties and not give up easily. Based on this, the level of argumentation ability according to McNeill & Krajcik's theory in SS subjects who have moderate productive struggle is presented in Table 5.

Table 5. Subject SS's Level of Argumentation Ability

No	Aspect	Description	Level
1.	Claim	SS tend to be able to make claims, but the claims made are incomplete and inaccurate.	1
2.	Evidence	SS can give evidence on the basis of data and information. This evidence alone, which is incomplete, supports the claim.	1
3.	Reasoning	SS tend to be able to provide reasons in the form of calculation steps, but there are errors that make them insufficient to support the claim.	1
4.	Rebuttal	SS do not make counterclaim. SS tend to be less able to provide sufficient and appropriate evidence and reasons to reject counterclaims.	0

3.1.3. Subject with Low Productive Struggle

Figure 5 shows that subject SR only rewrote question number 1. Subject SR was unable to write down the information that was known and asked in the question, so subject SR was unable to complete questions 1 and 2. Subject SR did not complete all required components of mathematical argumentation skills when he solved system of linear equations in three variables problems. The subject SR expressed through further interviews that he had no comprehension of system of linear equations in three variables material. The SR subject did not understand the system of linear equations in two variables material, and he remained confused about positive and negative number operations. The SR subject also did not dare to ask questions to the teacher or friends if there was material or questions that he did not understand because he was embarrassed. The SR subject also tended to be unmotivated to solve mathematical problems and stopped trying if he thought the question was difficult to do. The data emerging from the test and interviews assumes that SR, who undergoes rather an unproductive struggle, is a poor arguer or seems to have very poor argumentation ability. Based on this, the level of argumentation ability according to McNeill & Krajcik's theory in subject SR, who has low productive struggle, is presented in Table 6.

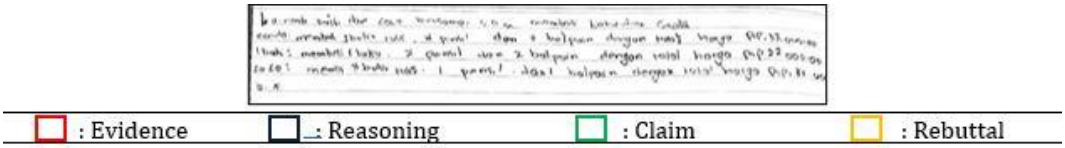

Figure 5. Subject SR's Answers to Equation Number 1

Table 6. Subject SR's Level of Argumentation Ability

No	Aspect	Description	Level
1.	Claim	SR cannot make a claim from the given problem.	0
2.	Evidence	SR provides data and information but is heavily against data being used in support of specific claims.	0
3.	Reasoning	SR cannot provide reasons in the form of calculation steps from the evidence to support the claim.	0
4.	Rebuttal	SR cannot make a counterclaim and cannot provide evidence and reasoning to refute the counterclaim.	0

3.2. Discussion

Based on the research results presented, a consistent pattern emerges, the higher the productive struggle of students, the more complete and stronger their mathematical argumentation ability. Students who experienced productive struggle at its highest level developed their ability to create arguments which contained distinct claims, suitable evidence, and logical reasoning. This occur because high productive struggle fosters students' willingness to endure cognitive conflict, giving them time to evaluate multiple strategies and verify

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each step before concluding. This sustained mental effort enables them construct not only claim but also reasoning that explicitly connects evidence to claims. This finding indicates that productive struggle support not only procedural understanding, but also deeper cognitive processes involved in constructing mathematical arguments. According McNeill and Krajcik (2011), a high-quality argument must include reasoning to justify how evidence support the claim, and rebuttal to account for alternative explanations. Conversely, students with low productive struggle produce lower levels of argumentation ability. This pattern shows that argumentation ability are not only the result of mastery of mathematical procedures but are also influenced by perseverance, students' readiness to struggle when faced with difficult problems, and their readiness to keep trying (Young et al., 2023). According to Yu et al. (2021), productive struggle helps students develop advanced reasoning skills because it requires them to assess their strategies and fix their errors while working on difficult tasks. In this material on system of linear equations in three variables, students with high productive struggle showed a similar pattern of checking, correcting, and confirming the steps of the solution so that they could produce valid arguments.

Students in the moderate productive struggle category produce partial arguments. Students can write down some claims and evidence, but the reasoning shown does not or insufficiently supports the claims appropriately. This occur because student at this level engage in struggle but stop when cognitive demand becomes too high. They have the initial motivation to start yet lack the metacognitive control to persist reasoning. As a result, the reasoning component becomes insufficiently developed within their argument structure. This situation is in line with Sinha & Kapur (2021) who found that students with moderate perseverance tend to be able to start an argument but have difficulty connecting evidence with reasoning using mathematical principles consistently. McNeill and Krajcik (2011) assert that without explicit reasoning, the justification for why evidence support a claim remains unclear, thus the argument is considered incomplete even when data is present. In addition, the partial level of mathematical argumentation of students with moderate productive struggle is influenced by their tendency to be hesitant and lack confidence in the steps they take, as well as their reluctance to ask questions. Datu et al. (2024) argues that perseverance is not only about persisting with every task given, but also the ability to monitor and assess the tasks they do themselves, which seems to be lacking in the students in this study.

Meanwhile, students in the low productive struggle category showed an inability to construct mathematical arguments. Students were unable to write down important information, unable to make claims, and preferred to stop before thinking more deeply. This condition occurs because students tend to avoid cognitive struggle and discontinue their efforts when encountering difficulties. This perspective is in agreement with the finding of Witherspoon et al. (2022), who have shown that students who struggle with a task have a tendency to avoid the struggle, not pushing themselves into the productive stage of persevering. According to McNeill and Krajcik (2011), a claim serves as the foundational component of an argument, if students fail to make a claim, the subsequent components of evidence and reasoning cannot be developed. Students prefer to stop trying when they encounter obstacles early on, preventing them from constructing arguments.

3.3. Implications

The research results of this study show several significant implications which require explanation. The design of mathematics instruction must enable students to face increasingly difficult challenges because this method helps them develop complete mathematical arguments through their academic work. The extent of productive struggle which students experience directly affects their ability to think mathematically. Teachers must establish a classroom atmosphere which helps students develop their determination while they face educational challenges. The study establishes a new theoretical framework which connects mathematical argumentation to productive struggle during problem-solving activities. The study shows that students' ability to create arguments depends on their mental abilities and their determination to keep working. These discoveries provide a reference point for upcoming mathematics education research which will explore how cognitive and emotional elements impact the learning process.

3.4. Limitations

This research contains multiple limitations which need to be examined. The research studied 33 students from a senior high school located in Garut Regency, West Java, Indonesia which restricted its geographical reach and its number of participants. This makes it difficult to generalize the results to a wider population because it does not reflect the diversity of student characteristics as a whole. Second, the qualitative approach using case study methods was not designed to analyze the causal relationship between the variables studied. Third, the number of test questions, which was only two items, was also a limitation of this study because it was not sufficient to comprehensively evaluate students' mathematical argumentation ability. Based on these limitations, further research is recommended to involve a large and diverse sample, covering various school levels and different geographical locations, so that the research results will be more representative. Future researchers may consider using a mixed method that combines qualitative and quantitative approaches to achieve better understanding of how the research variables affect each other. The measurement tools need to

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be expanded in terms of both the number and variety of questions, so that they can describe various aspects of students' argumentation ability more accurately.

4. Conclusion

The study results demonstrate that productive struggle directly influences students' ability to make arguments about system of linear equations in three variables. Students who maintain high levels of productive struggle can produce complete argumentation which includes all essential components of claims evidence reasoning and rebuttals. Students with moderate productive struggle show inconsistent partial argumentation ability while students with low productive struggle tend to be unable to construct arguments due to limited conceptual understanding and a lack of persistence in facing challenges. These findings indicate that argumentation ability is not only influenced by cognitive aspects, but also by students' resilience, effort, and readiness to engage in the problem-solving process.

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All authors have equal contributions to the paper. All the authors have read and approved the final manuscript.

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Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/ or publication of this article.

Data Availability

The datasets generated during and/ or analyzed during the current study are available from the corresponding author on reasonable request.

Declaration on AI Use

The author(s) declare that no artificial intelligence (AI) or AI-assisted tools were used in the preparation of this manuscript.

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